The University of Texas at Austin
Dept. of Electrical and Computer Engineering
Midterm #1

Date: October 2, 2003

Course: EE 313 Evans/Arifler

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Last, First

- The exam is scheduled to last 75 minutes.
- Open books and open notes. You may refer to your homework assignments and the homework solution sets.
- Calculators are allowed.
- You may use any standalone computer system, i.e. one that is not connected to a network.
- All work should be performed on the quiz itself. If more space is needed, then use the backs of the pages.
- **Fully justify your answers unless instructed otherwise.**

Problem	Point Value	Your score	Topic
1	20		Differential Equation
2	20		Discrete-Time System Response
3	20		Tapped Delay Line
4	24		Continuous-Time System Responses
5	16		Potpourri
Total	100		

K-24

Problem 1.1 Differential Equation. 20 points.

For a continuous-time system with input $f(t)$ and output $y(t)$ governed by the differential equation

$$\frac{d^2}{dt^2}y(t)+7\frac{d}{dt}y(t)+6y(t)=f(t)$$

(a) What are the characteristics roots of the differential equation? 4 points.

The characteristic polynomial is $\lambda^2+7\lambda+6$.
Its roots are $-1, -6$: $(\lambda+1)(\lambda+6)=0$

(b) Find the zero-input response assuming non-zero initial conditions. Please leave your answer in terms of C_1 and C_2. 8 points.

$y(t) = C_1 e^{-t} + C_2 e^{-6t}$ for $t \geq 0^+$.

$y(0) = C_1 + C_2$

$y'(0) = -C_1 - 6C_2$

because $y'(t) = -C_1 e^{-t} - 6C_2 e^{-6t}$

(c) Find the zero-input response for the initial conditions $y(0^+) = 5$and $y'(0^+) = 0$. 8 points.

$C_1 + C_2 = 5$

$-C_1 - 6C_2 = 0$ $\Longrightarrow$ adding the two equations,

$-5C_2 = 5 \Rightarrow C_2 = -1$

So $C_1 = 6$

$y(t) = 6e^{-t} - e^{-6t}$ for $t \geq 0^+$

K-25

Problem 1.2 Discrete-Time System Response. 20 points.
A discrete-time linear time-invariant system has the impulse response

$$h[k] = \left(\frac{1}{2}\right)^k u[k]$$

By any means necessary, find the output $y[k]$ for

(a) an input of

$$f[k] = \left(\frac{1}{2}\right)^k u[k]$$

$$y[k] = h[k] * f[k] = \left(\frac{1}{2}\right)^k u[k] * \left(\frac{1}{2}\right)^k u[k]$$
$$= (k+1)\left(\frac{1}{2}\right)^k u[k]$$

using Table 3.1, line 8, in Lathi's book, page 217.

(b) an input of a rectangular pulse

$$f[k] = \begin{cases} 1 & \text{for } 0 \le k \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

$$f[k] = u[k] - u[k-N]$$
$$y[k] = h[k] * f[k] = \left(\frac{1}{2}\right)^k u[k] * \left(u[k] - u[k-N]\right)$$
$$= \left(\frac{1}{2}\right)^k u[k] * u[k] - \left(\frac{1}{2}\right)^k u[k] * u[k-N]$$
$$= \underbrace{\frac{1-\left(\frac{1}{2}\right)^{k+1}}{1-\frac{1}{2}} u[k]}_{\text{from Table 3.1, line 2, in Lathi}} - \underbrace{\frac{1-\left(\frac{1}{2}\right)^{k-N+1}}{1-\frac{1}{2}} u[k-N]}_{\text{from using the shifting property of convolution applied to the convolution in line 2 of table entry in Lathi Table 3.1}}$$

K-26

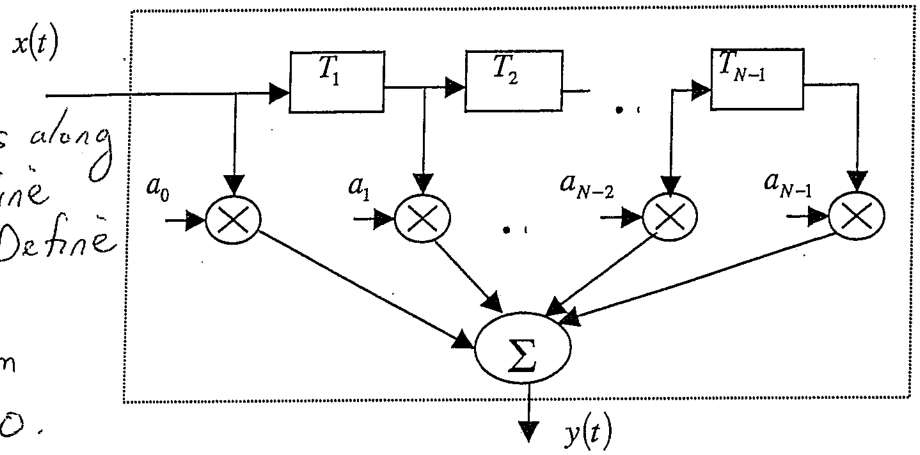
Problem 1.3 Tapped Delay Line. 20 points.

A continuous-time tapped delay line, in which each of the N-1 delay blocks has a possibly different delay value, is shown below as a block diagram:

The delays along the delay line add up. Define

$$\tau_n = \sum_{m=0}^{n} T_m$$

with $T_0 = 0$.

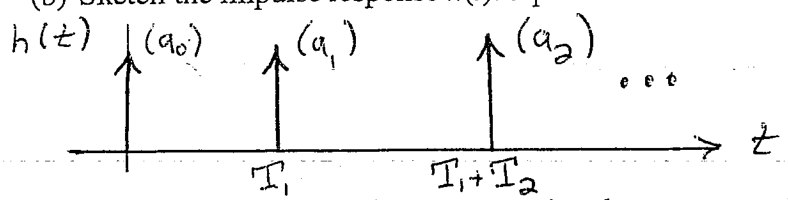


(a) Give a formula for the impulse response $h(t)$. 5 points.

Let $x(t) = \delta(t)$. $h(t) = a_0\,\delta(t) + a_1\,\delta(t - T_1) + a_2\,\delta(t - T_1 - T_2) + \cdots$

$$h(t) = \sum_{n=0}^{N-1} a_n\,\delta(t - \tau_n)$$

(b) Sketch the impulse response $h(t)$. 5 points.



(c) Give a formula for the step response, i.e. the response when the unit step $u(t)$ is input. 5 points.

Let $x(t) = u(t)$. $y_{step}(t) = \sum_{n=0}^{N-1} a_n\,u(t - \tau_n)$

(d) What is the system time constant? 5 points.

Since the delays along the delay line add up, the system time constant is $\tau_{N-1} = \sum_{m=0}^{N-1} T_m$, where $T_0 = 0$. The system time constant is the non-zero duration of the impulse response, since the impulse response has finite extent.

Problem 1.4 Continuous-Time System Responses. 24 points.

Consider the continuous-time linear time-invariant system with input $f(t)$ and output $y(t)$ that is an ideal differentiator shown on the right:

$$f(t) \longrightarrow \boxed{\frac{d}{dt}(\bullet)} \longrightarrow y(t)$$

$$y(t) = \frac{d}{dt}(f(t))$$

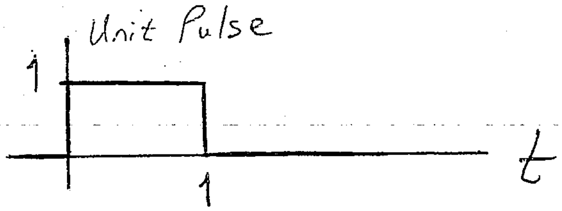
(a) What is the impulse response? 8 points.

Let $f(t) = \delta(t)$. $\quad h(t) = \frac{d}{dt}(\delta(t)) = \delta'(t)$

(b) What is the (unit) pulse response? 8 points.

Let $f(t) = u(t) - u(t-1)$.

$$y_{pulse}(t) = \frac{d}{dt}\left(u(t) - u(t-1)\right) = \delta(t) - \delta(t-1)$$



(c) What is the (unit) step response? 8 points.

Let $f(t) = u(t)$.

$$y_{step}(t) = \frac{d}{dt}(u(t)) = \delta(t)$$

Problem 1.5 Potpourri. 16 points.

(a) Derive the shifting property of the impulse signal using the convolution definition

i. In continuous time. $f(t) * \delta(t - t_0) = f(t-t_0)$. 4 points.

$$f(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} f(t-\tau)\,\delta(\tau-t_0)\,d\tau$$

We know that $\delta(x) = 0$ when $x \neq 0$, and $\int_{-\infty}^{\infty} \delta(x)\,dx = 0$. $\delta(\tau - t_0)$ is non-zero only at $\tau = t_0$. It has unit area at $\tau = t_0$.

$$f(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} f(t-\tau)\,\delta(\tau-t_0)\,d\tau = f(t-t_0)$$

ii. In discrete time: $f[k] * \delta[k - k_0] = f[k - k_0]$. 4 points.

$$f[k] * \delta[k-k_0] = \sum_{m=-\infty}^{\infty} f[k-m]\,\delta[m-k_0]$$

We know that $\delta[x] = 0$ when $x \neq 0$. $\delta[m-k_0]$ is non-zero (has value of 1) only at $m - k_0 = 0 \Rightarrow m = k_0$.

$$f[k] * \delta[k-k_0] = \sum_{m=-\infty}^{\infty} f[k-m]\,\delta[m-k_0] = \sum_{m=k_0}^{k_0} f[k-m] = f[k-k_0]$$

(b) Give one signal processing or communication system that uses each of the following subsystems and describe the role that the subsystem plays in the function of the overall system:

i. Resonators. 4 points.

A resonator is designed to have its largest/strongest response to match an certain input signal. The impulse response of an LTI system is designed to match the certain input signal. Resonators are used in all kinds of comm. receivers/decoders - touchtone detectors, AM/FM demodulators, etc.

ii. Oscillators. 4 points.

An oscillator could be a sinusoidal generator. Sinusoidal generators are used to generate carrier frequencies for AM/FM radio, cell phones, broadcast TV, etc. Sinusoidal generators ~~are used to generate~~ are used to generate telephone touchtones signals, which are a sum of two sinusoids.

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